Chapter 1
Introduction to input-output analysis

1.1. Overview

The most transparent model of a supplier-purchaser relationship is the one proposed by Wassily Leontief. The model, inspired by works of François Quesnay and Leon Walras, will be used as the basis for further considerations in this book. Its essence boils down to the assumption that the national economy is an aggregate of resources and streams that jointly form several related systems that are all described in the framework of a tabular input-output approach and that control production, services, international trade with foreign countries, households, budget, and banks. Leontief model was first mainly developed by his associates and students. In the 1950s, input-output regional models were created by Isard, Chenery, and Moses, while Dorfman, Samuelson, and Solow expanded Leontief's approach using optimization techniques. Also, Leontief published several famous results of research on the share of production factors in foreign trade, giving a strong boost to the search for new theories of foreign trade. In the 1960s, European economists came to the fore; one of them – Richard Stone – laid the ground for establishing the System of National Accounts (SNA), which were aimed at providing a comprehensive conceptual and accounting framework for compiling and reporting macroeconomic statistics for analyzing and evaluating the performance of an economy. The 1970s brought the development of dynamic input-output models and global trade models. In 1973, Leontief was honored with the Nobel Prize in economics. At that time, macroeconomic models based on input-output tables (integrated and CGE models) gained the formal mathematical forms that are being used today (Tomaszewicz, 1983). In the next decade, social accounting matrices (SAMs) were developed by Stone in Great Britain, who was also a Nobel Prize laureate (he received the prize in 1984).

Since this book is not a general textbook on input-output modeling, the content of this chapter is not aimed at providing a complete course of training. The reader may find excellent and extensive didactic materials on IO modeling in the famous textbook by Miller and Blair (2009). However, a brief introduction into the general

assumptions of this type of economic analysis seems required to better understand the major content of the book, which is concentrated around the issue of key sector analysis and the identification of ICs.

1.2. National input-output tables

Input-output analysis is based on an examination of the flows (usually expressed in monetary terms) of products from each sector that is considered to be a producer to each sector (itself and others) that is considered to be a consumer. As already mentioned, this basic information (which is required to construct an input-output model) is contained in an interindustry transactions table. A general theoretical scheme of national IO table (with n sectors) is presented in Figure 1.1.

		Producers as consumers			Final demand			
		Sector 1	...	Sector n	Personal consumption expenditures	Gross private domestic investment	Govt. purchases of goods and services	Net exports of goods and services
Producers	Sector 1							
	...							
	Sector n							
Value added	Employees	Employee compensation			Gross domestic product			
	Business owners and capital	Profit-type income and capital consumption allowances						
	Government	Indirect business taxes						

Figure 1.1. General theoretical structure of national IO table

Source: Own elaboration based on Miller and Blair (2009).

The rows of such a table represent the distribution of a producer's output across the economy, while the columns describe the composition of the inputs required by a particular industry to produce its output (Miller and Blair, 2009). The interindustry flows of goods constitute the shaded portion of the table presented in Figure 1.1. The columns labeled as 'Final demand' present the information on the sales by each sector to final markets for their production, such as personal consumption purchases and sales to the government[6]. On the other hand, the rows labeled as 'Value added' represent other (non-industrial) inputs to production (e.g., labor), depreciation of capital, indirect business taxes, and imports.

Since the scope of the empirical part of this book is focused on Polish economy, in Figure 1.2 I will also present a general structure of a national IO table provided by Central Statistical Office (CSO) in Poland.

[6] For example, alcohol may be sold to businesses in other sectors as a required chemical input to production (this is an example of an interindustry transaction) and also to consumers (this, in turn, is an example of a final-demand sale).

Uses / Products (CPA)	Intermediate consumption (CPA)			Final demand											Total use at basic prices
				final consumption expenditure				gross capital formation			exports fob			total	
	Sector 1	...	Sector n	by households	by non-profit institutions serving households (NPISH)	by government	total	gross fixed capital formation	changes in inventories and changes in valuables	total	intra European Union	extra European Union	total		
Sector 1															
...															
Sector n															
Total products															
Taxes less subsidies on products															
Total intermediate consumption/final demand at purchasers' prices															
Compensation of employees Other net taxes on production Consumption of fixed capital Operating surplus, net Operating surplus, gross															
Value added at basic prices															
Output at basic prices															
Imports cif															
Supply at basic prices															

Figure 1.2. General structure of national IO table (with n sectors) provided by Central Statistical Office of Poland

Note: 'CPA' is an acronym for the **C**lassification of **P**roducts by **A**ctivity. The value of imports is shown in terms of cif Polish port or franco Polish border including transport and insurance costs to the Polish border. The value of export is shown on the fob basis, that is in terms of franco border or fob port of the country of delivery.

Source: Own elaboration based on Przybyliński (2012) and Central Statistical Office of Poland (2014).

The most recent IO table available at the time of preparing this book was published by the CSO in 2014 and reflected the interindustry relationships in the Polish economy in 2010[7]. The table presents data on the flows at basic prices in a product-by-product layout. Similarly to Figure 1.1, the interindustry exchanges of goods constitute the shaded portion of the table presented in Figure 1.2. As can be seen in Figure 1.2, the CSO in Poland distinguishes three general parts of the national input-output table: intermediate consumption matrix; final demand matrix with a division into specific components (i.e., final consumption expenditure by households, non-profit organizations serving houses and government, gross fixed capital formation, changes in inventories and valuables, and exports); and value added matrix containing data on compensation of employees, other net taxes on production, consumption of fixed capital, net operating surplus, and gross operating surplus. The input-output tables published by the CSO in Poland are elaborated on the basis of the Polish Classification of Goods and Services[8].

The data on the flows of goods and services in the national input-output table published by the CSO in Poland is expressed in the basic prices obtained as a result of subtracting the taxes less subsidies on the products and trade as well as transport margins from any flow at purchasers' prices[9].

1.3. Construction of WIOD

In order to empirically analyze the trends in international production fragmentation and their impact on global trade, one needs to have access to a consistent time-series of world input-output tables. In recent years, the IO field has gained a new impetus due to the development of the World Input-Output Database.

The first version of the World Input-Output Database was constructed within the official WIOD Project, which was funded by the European Commission as part of the Seventh Framework Program[10]. Last updated in November 2016, the database

[7] The 2010 input-output table for Poland provides data on $n = 77$ sectors (Central Statistical Office of Poland, 2014).

[8] The 2010 IO table provided by the CSO in Poland was based on a sectoral classification reported in the Polish Classification of Goods and Services of 2008. The structure of the classification is based on the Statistical Classification of Economic Activities in the European Community (this is officially referred to as 'NACE' due to the French translation: ***N**omenclature statistique des **a**ctivités économiques dans la **C**ommunauté **e**uropéenne*), the Classification of Products by Activity (CPA), and the PRODCOM List. For more details see: `http://stat.gov.pl/en/metainformations/classifications`.

[9] In addition to the table depicted in Figure 1.2, CSO of Poland also publishes input-output tables at basic prices for domestic output. These tables have analogical layout like the IO table in Figure 1.2, however the data on use of imported products is included in intermediate consumption, while rows of imports cif and supply at basic prices are deleted. In the input-output table at basic prices for domestic output data concerning flows of goods and services is worked out by subtracting flows of imported goods and services from flows of goods and services at basic prices. The value of flows of imported goods and services are shown at row of use of imported products (Central Statistical Office of Poland, 2014).

[10] The WIOD project was funded between May 2009 and April 2012 by the Seventh Framework Program Theme 8: Socio-Economic Sciences and Humanities. The database was officially launched on April 16, 2012, in Brussels during a high-level conference on *Competitiveness, trade, environment, and jobs in Europe: Insights from the new World Input Output Database* (*WIOD*). For more details on the history of WIOD, visit `http://www.wiod.org/project`.

		Block I: Intermediate demand				Block II: Distribution of final demand				
		Country 1 intermediate use INDUSTRIES	Country 2 intermediate use INDUSTRIES	...	Country C intermediate use INDUSTRIES	Country 1 final domestic use	Country 2 final domestic use	...	Country C final domestic use	Output
Country 1	INDUSTRIES	Intermediate use of domestic output by Country 1	Intermediate use by Country 2 of imports from Country 1	...	Intermediate use by Country C of imports from Country 1	Final use of domestic output by Country 1	Final use by Country 2 of imports from Country 1	...	Final use by Country C of imports from Country 1	Output in Country 1
Country 2	INDUSTRIES	Intermediate use by Country 1 of imports from Country 2	Intermediate use of domestic output by Country 2	...	Intermediate use by Country C of imports from Country 2	Final use by Country 1 of imports from Country 2	Final use of domestic output by Country 2	...	Final use by Country C of imports from Country 2	Output in Country 2
...		...	...	...	...	...	...	...	...	...
Country C	INDUSTRIES	Intermediate use by Country 1 of imports from Country C	Intermediate use by Country 2 of imports from Country C	...	Intermediate use of domestic output by Country C	Final use by Country 1 of imports from Country C	Final use by Country 2 of imports from Country C	...	Final use of domestic output by Country C	Output in Country C
Block III: Income		Taxes less subsidies Cif/ fob adjustments Direct purchases abroad by residents Purchases on the domestic territory by non-residents, Value added at basic prices Transport margins	Taxes less subsidies Cif/ fob adjustments Direct purchases abroad by residents Purchases on the domestic territory by non-residents, Value added at basic prices Transport margins	...	Taxes less subsidies Cif/ fob adjustments Direct purchases abroad by residents Purchases on the domestic territory by non-residents, Value added at basic prices Transport margins					
		Output in Country 1	Output in Country 2	...	Output in Country C					

Figure 1.3. General structure of WIOT provided by World Input-Output Database

Note: Similar to national IO tables (comp. Figures 1.1 and 1.2), the final domestic use in each country is divided into the following subcategories: final consumption expenditure by households; final consumption expenditure by non-profit organizations serving households (NPISH); final consumption expenditure by government; gross fixed capital formation; and changes in inventories and valuables.

Source: Own elaboration based on Timmer et al. (2016).

contains the time series of global inter-country input-output tables assembled from national accounts data, supply-use tables, and data on international trade in goods and services (Timmer et al., 2015; Timmer et al., 2016). In other words, one may treat a world input-output table (WIOT) as a set of national input-output tables that are linked with each other by bilateral international trade flows. It is obvious that this type of combination of the domestic and international flows of products opens a way for conducting formal analysis of global production networks that could not have been carried out earlier.

Because of the integration of statistics across countries, the general structure of the WIOTs provided by the World Input-Output Database is very similar to that of national IO tables, which are routinely produced by national statistical institutes. Since many empirical applications require a square matrix that reflects economic linkages across industries in different countries, the WIOTs are published in an industry-by-industry format. In the process of constructing WIOTs, national supply and use tables that contain data on industries and products are used. A general structure of the WIOTs published by WIOD is presented in Figure 1.3. The products (industries) are classified according to the classification of products by activity (CPA) and cover 56 product categories following the primary outputs from the 56 sectors (industries)[11]. Similar to the national IO tables (comp. Figure 1.2), the WIOTs are also divided into three general parts: Bloc I (intermediate consumption matrix); Bloc II (final demand matrix); and Bloc III (taxes less subsidies, cif/fob adjustments on exports, direct purchases abroad by residents, purchases on the domestic territory by non-residents, value added at basic prices, and international transport margins).

The new 2016 release of WIOD provides access to annual time-series of WIOTs for 43 countries, including all 28 members of the European Union and 15 other major economies. The database covers the period of 2000−2014. In general, the choice of the countries listed in WIOD follows from two facts. First of all, one must take into account the issue of data availability of a sufficient quality. Second, the choice of the countries should ensure that a major part of the world economy is covered in the interindustry relationships in the global IO table[12].

Before one can assess the usefulness of WIOD for the analysis of global trade links, it is important to understand the basic construction approach. Figure 1.4 presents the main stages of the process of constructing the WIOTs. In brief, WIOTs are constructed in several steps. First, using the SUT-RAS method (Temurshoev and Timmer, 2011), time-series of supply-use tables (SUTs) at purchasers' prices are generated. This stage requires the extrapolation and benchmarking of the SUTs to national accounts statistics.

At the next stage, the time-series of SUTs at purchasers' prices are transformed into SUTs in basic prices. This stage in turn requires the construction of international trade and transport margins and net taxes at the product level, and then estimating

[11] Secondary outputs of industries are accounted for in the supply tables.

[12] Together, the 43 countries listed in WIOD cover more than 85 percent of world GDP at the current exchange rates (Timmer et al., 2016). In addition, a model for the remaining non-covered part of the world economy is estimated; this is called the *Rest of the World* region. Since the set of countries called *Rest of the World* in the WIOD database is added for balancing and calculation purposes only (Dietzenbacher et al., 2013b) and serves as a proxy for countries not included in the sample, this group of economies is not amenable to interpretation (Timmer, 2012).

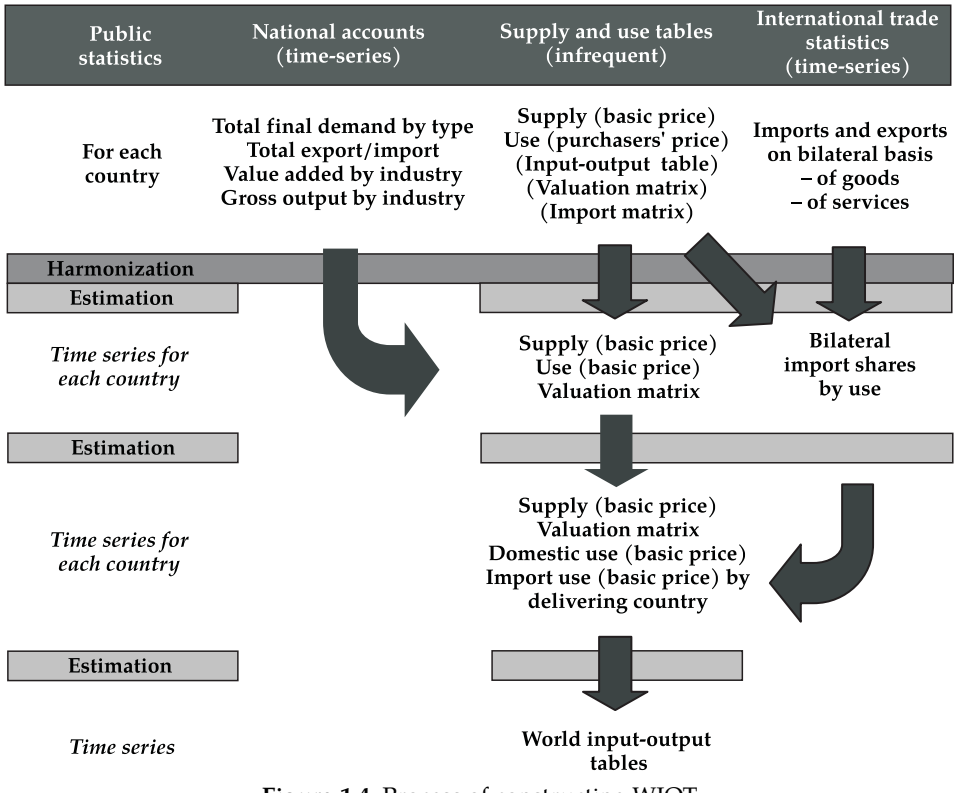


Figure 1.4. Process of constructing WIOT

Source: Own elaboration based on Timmer et al. (2016).

the valuation matrices based on the structure in the margins. At the third stage, the national SUTs are transformed into inter-country SUTs by breaking down the use table into domestic and imported (by delivering country) components and using the data on imports from the international trade statistics. The final stage is aimed at moving from the SUTs to inter-country input-output tables[13]. It is important to underline that, in addition to the WIOTs in current prices, WIOD offers access to global IO tables expressed in constant prices. The procedure of obtaining WIOTs in constant prices is based on the application of industry output deflators, which are used to conduct the row-wise deflation of the SUTs (Dietzenbacher et al., 2013b).

At this point, one may ask a question on comparing the WIOD database with competing databases on global trade interrelations. Among the possible alternatives, the Eora database seems to be the most common choice (Lenzen et al., 2012; Lenzen et al., 2013)[14]. When compared to WIOD, the global IO tables delivered by Eora are available for a larger number of countries and span over

[13] It is important to note that exports to the group of the *Rest of the World* countries is calculated as residual and can become negative (Timmer et al., 2016).

[14] See Tukker and Dietzenbacher (2013) for an excellent overview of existing global input-output databases.

a longer time period. However, the alternative cost is that, in contrast to WIOD, the WIOTs available in Eora are not constructed under the requirement of a strict hierarchy in the basic data sources, but the process of constructing a single WIOT starts from an initial situation in which all of the available basic information is incorporated[15]. As underlined by Timmer et al. (2016), such an approach leads to the fact that Eora tables do not adhere to national accounts statistics. Another problem is that a large number of elements in the WIOTs published in the Eora database show a tendency to fluctuate between years, which in turn precludes detailed analyses. On the contrary, the process of constructing the WIOTs in WIOD is conducted in the framework of the most recent SNA; thus, all concepts and accounting identities in the SNA are taken into account. This also refers to the issue of dealing with conflicting information such as breaks in respective time series or differences between levels of trade in national and international statistics; such issues are resolved in the framework of the SNA before a respective optimization problem is solved. All of these features allow for the claim that the WIOTs in WIOD are characterized with a relatively high level of data quality and maximal consistency, which in turn may prove supreme over competing databases on global trade links.

When discussing the properties of WIOD, one cannot omit the major disadvantages and limitations of the approach. One of the critical issues in this context is the lack of any uncertainty measures around the estimates provided in WIOD. In general, IO analysis remains one of the few analytical tools that does not explicitly present error bounds in presenting the tables. Yet, many of the inter-country parts of the tables have been estimated with a combination of survey and non-survey techniques, while most of the WIOD system is estimated for national income and product accounts. Having a sense of the possible errors that may result from such an approach might help to place each empirical analysis based on WIOD data in a more appropriate context.

As already mentioned, using disaggregated WIOD data results in an evaluation of the time series of global IO tables – each with around six million elements. In some applications (especially those aimed at analyzing general trends in global trade), such large tables may seem to be too big to be discussed in full detail in a single study. Thus, a typical strategy of conducting empirical research in such a context is to use aggregated variants of the global IO tables. However, one cannot forget that any type of aggregation of the original IO data may lead to undesirable effects; this is referred to as 'aggregation bias' in the literature. As underlined by Morimoto (1970), total aggregation bias may be defined as the difference between the vector of outputs in an aggregated system and the vector obtained by aggregating the outputs in the original disaggregated system. As underlined by Kymn (1990), this type of bias may often reach relatively high levels; thus, an interpretation of the results of any empirical analysis based on aggregated IO data should always take this problem into account.

[15] When conflicting pieces of information are found in the data, the solution is based on simply attaching the respective measures of reliability and running an optimization algorithm to distribute the differences across all other cells in the global IO matrix (Lenzen et al., 2012; Lenzen et al., 2013).

Finally, one cannot forget that the majority of international organizations that deal with constructing international IO data (including WIOD) use foreign exchange rates for measuring and comparing the production of different nations. However, the rates of foreign exchange result from the activity of financial markets dealing with property assets rather than labor products (Reich, 2018); as a consequence, they are incomparable with the national prices that are formed on national product markets operating within the realm of a well-defined and homogeneous national currency as its unit of measurement. As a result, using foreign exchange rates as a measure of value may be the source of an unwarranted bias that can lead to the false interpretation of changes in national conditions of production. Alternatively, using international purchasing power parities (PPP) opens the way to separating the effects of the currency exchange rates from the effects of the conditions of production reflected in the national prices; thus, this helps deepen and clarify the analysis of value added chains. Although it is clear that PPP-conversion is superior to using nominal exchange rates, the main obstacle for recalculating global IO tables in PPP is the lack of reliable international statistical data. Therefore, it is not surprising that conducting this type of conversion is listed among the main goals for the future development of WIOD (Timmer et al., 2016).

1.4. Basic Leontief input-output model

1.4.1. Single-economy case

Following the usual notation in the IO literature, matrices will be indicated by bold capital letters, vectors by bold lowercase letters, and scalars by italic capital and lowercase letters throughout the following chapters of this book. Transposition is indicated by a prime symbol, and a circumflex denotes a diagonal matrix (for example, $\hat{\mathbf{x}}$ has elements of vector $\mathbf{x}$ on the main diagonal, and $\hat{\mathbf{x}}^{-1}$ denotes a diagonal matrix with the inverses of the elements of nonzero vector $\mathbf{x}$ on the main diagonal)[16].

In order to derive the basic linear form of the static Leontief model, let us assume that the economy under study consists of n productive sectors, and the respective data is available in year t. If x_i^t denotes the output[17] of sector i and f_i^t stands for the total final demand for sector i's product for period t[18], one may write a simple equation explaining how sector i's product is distributed through sales to all sectors in the economy and to final demand:

$$x_i^t = z_{i1}^t + \ldots + z_{in}^t + f_i^t = \sum_{j=1}^{n} z_{ij}^t + f_i^t, \tag{1.1}$$

where z_{ij}^t represents the value of the flow of goods and services that were produced in sector i in the economy under study and consumed in sector j in year t.

[16] Throughout this book, I will also use $\text{diag}(x_j, j = 1, \ldots, n)$ to refer to a diagonal matrix with elements x_j on the main diagonal.

[17] Throughout this book, I will use the terms 'output' and 'gross output' interchangeably.

[18] Throughout this book, I will use the terms 'period' and 'year' interchangeably to denote the time interval of interest.

One can easily combine the accounting formulas in (1.1) across all sectors and obtain the following compact matrix formula:

$$\mathbf{x}_t = \mathbf{Z}_t\mathbf{i} + \mathbf{f}_t, \tag{1.2}$$

where:

$$\mathbf{x}_t = \begin{bmatrix} x_1^t \\ \vdots \\ x_n^t \end{bmatrix}, \quad \mathbf{Z}_t = \begin{bmatrix} z_{11}^t & \cdots & z_{1n}^t \\ \vdots & \ddots & \vdots \\ z_{n1}^t & \cdots & z_{nn}^t \end{bmatrix}, \quad \mathbf{f}_t = \begin{bmatrix} f_1^t \\ \vdots \\ f_n^t \end{bmatrix} \tag{1.3}$$

and $\mathbf{i}$ denotes a $n \times 1$ vector of 1's[19]. A fundamental assumption in static IO analysis is that the interindustry flows from sector i to sector j at period t depend entirely on the output of sector j for the same time period. This property is expressed in the following definition of the so-called 'technical coefficients':[20]

$$a_{ij}^t = \frac{z_{ij}^t}{x_j^t}, \tag{1.4}$$

where $i, j = 1, \ldots, n$. The elements a_{ij}^t are viewed as the measures of fixed relationships between a sector's j output and its inputs. Thus, economies of scale in production are ignored – production in a Leontief system operates under which is known as constant returns to scale (Miller and Blair, 2009). For example, if sector i stands for a sector of textiles and sector j stands for automotive sector, a_{ij}^t represents the ratio of the value of the textile goods bought by automotive producers in year t to the value of the automotive production in year t.

An important characteristic of a static Leontief input-output model is the assumption of the fixed proportions of the use inputs. Using (1.4) and some trivial algebra, one may simply write:

$$z_{ij}^t = a_{ij}^t x_j^t, \quad \text{where } i, j = 1, \ldots, n, \tag{1.5}$$

which in turn implies:

$$\frac{z_{ij}^t}{z_{sj}^t} = \frac{a_{ij}^t x_j^t}{a_{sj}^t x_j^t} = \frac{a_{ij}^t}{a_{sj}^t} = \text{constant, if only } a_{sj}^t x_j^t \neq 0. \tag{1.6}$$

Condition (1.6) implies that, for sector j, the proportions of the use inputs from sector i and sector s are equal to the corresponding ratios of the technical coefficients; since the technical coefficients are fixed, so are the ratios[21]. Using (1.4) and the formal definition of production functions (which relate the amounts of the inputs used by a sector to the maximum amount of output that could be produced by that

[19] For each sector i, final demand f_i^t is a sum of the final consumption expenditure by households, final consumption expenditure by non-profit organizations serving households (NPISH), final consumption expenditure by government, gross fixed capital formation, and changes in inventories and valuables in year t.

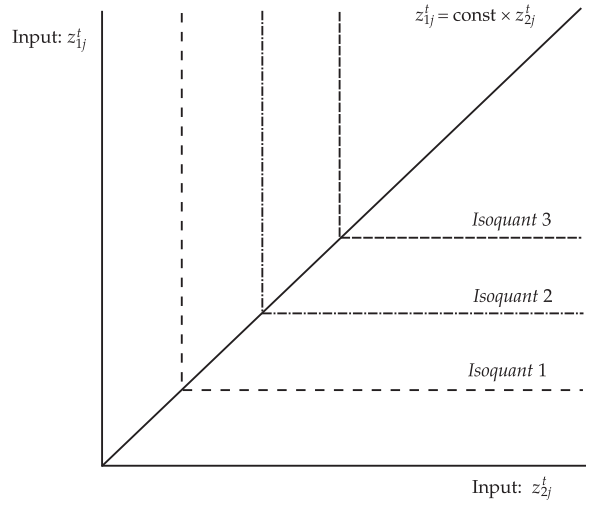
[20] In the input-output literature, the terms 'input-output coefficient' and 'direct input coefficient' are used interchangeably.

[21] The assumption of fixed proportions of use inputs also holds for different but close time periods. This follows from the fact that input-output coefficients are by and large stable in the short-term (comp. Carter [1970] and Pan [2006], for example).

sector with these inputs), one can simply provide a formula for the production function in the Leontief IO model:

$$x_j^t = \min\left(\frac{z_{ij}^t}{a_{ij}^t}, i = 1, \ldots, n\right) \text{ for } j = 1, \ldots, n. \tag{1.7}$$

To geometrically represent the Leontief production functions given in (1.7) in input space, one may consider a simple example of a two-sector economy[22]. As shown in Figure 1.5, the isoquants of the constant output take an L-shaped form in the case of such a model.



Production at *Isoquant* 3 > Production at *Isoquant* 2 > Production at *Isoquant* 1

Figure 1.5. Leontief production function in two-sector input space

Source: Own elaboration based on Miller and Blair (2009).

As implied by (1.6), the proportion of inputs remains constant; i.e., $z_{1j}^t / z_{2j}^t = \text{const}$. As a consequence, this will not bring any rise in the output of sector j if only one of the inputs rises but the other remains at its initial level. Only when both inputs are proportionally increased x_j^t may also increase. The simple two-sector example represented in Figure 1.5 may be easily generalized for the case of an n-sector economy. A general observation is that the Leontief production functions require inputs in fixed proportions, where a fixed amount of each input is required to produce one unit of output (Miller and Blair, 2009).

For example, assume that the inputs used in sector j in an n-sector economy were all tripled but the inputs from the last sector were only doubled. From the definition of the Leontief production function given in (1.7), one may then conclude that the minimum of the new ratios would be equal to 2; as a consequence, the new

[22] It is clear that a general geometric representation should deal with an n-dimensional input space with a separate axis for each of the n possible inputs. For dimensions greater than three, however, such a representation is impossible to visualize on a single plot; at the same time, the general principles are exactly the same as in the illustrative two-sector case.

output of sector j would be exactly twice as large. At the same time, there would be excess and unused amounts of inputs from all sectors except the last one.

One can easily combine the technical coefficients defined in (1.4) across all possible flows in an n-sector economy and obtain the following compact matrix formula:

$$\mathbf{A}_t = \mathbf{Z}_t \hat{\mathbf{x}}_t^{-1}, \tag{1.8}$$

where:

$$\hat{\mathbf{x}}_t = \begin{bmatrix} x_1^t & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & x_n^t \end{bmatrix}, \quad \mathbf{A}_t = \begin{bmatrix} a_{11}^t & \cdots & a_{1n}^t \\ \vdots & \ddots & \vdots \\ a_{n1}^t & \cdots & a_{nn}^t \end{bmatrix}. \tag{1.9}$$

Following the usual terminology in the IO literature, matrix $\mathbf{A}_t$ will henceforth be referred to interchangeably as the 'input matrix' or 'technology matrix'. Using (1.8), one may rewrite the set of accounting relationships in (1.2) in the following form:

$$\mathbf{x}_t = \mathbf{A}_t \mathbf{x}_t + \mathbf{f}_t, \tag{1.10}$$

or equivalently:

$$(\mathbf{I} - \mathbf{A}_t)\mathbf{x}_t = \mathbf{f}_t, \tag{1.11}$$

where $\mathbf{I}$ is an $n \times n$ identity matrix. The notation of the Leontief model given in (1.11) serves to make the dependence of the interindustry flows on the outputs of each sector explicit[23].

However, a different question is usually the case in practical applications of the static Leontief model; i.e., given the forecasts of the demands of the exogenous sectors, find the output from each of the sectors necessary to meet these forecasted final demands. If only $(\mathbf{I} - \mathbf{A}_t)^{-1}$ exists, this type of question may be easily answered using the following formula:

$$\mathbf{x}_t = (\mathbf{I} - \mathbf{A}_t)^{-1}\mathbf{f}_t = \mathbf{L}_t \mathbf{f}_t, \tag{1.12}$$

where matrix $\mathbf{L}_t = (\mathbf{I} - \mathbf{A}_t)^{-1} = [l_{ij}^t, i, j = 1, \ldots, n]$ is called the 'Leontief inverse'. In other words, the interindustry relationships in a given economy are analyzed from a demand-driven perspective in model (1.12). In this case, the Leontief inverse relates the sectoral gross outputs to the amount of the final product (final demand) – that is, to a unit of the product leaving the interindustry system at the end of the process (Panek, 2003; Miller and Blair, 2009)[24].

[23] Formally, the demand-driven Leontief input-output model consists of two major blocks of equations. In addition to the block of equations in (1.11), one should also consider the following equation: $\mathbf{tfu}_t = \mathbf{F}_t \mathbf{x}_t$, where $\mathbf{F}_t$ is the $k \times n$ matrix of factor inputs per unit of output (one row for each of k factors) at period t, and $\mathbf{tfu}_t$ stands for the vector of total factor use at period t (Duchin and Steenge, 2007).

[24] Let $\bar{\mathbf{f}}_t = [\bar{f}_s^t, s = 1, \ldots, n]$ correspond to a unit of final demand in sector j at period t, i.e.:

$$\bar{f}_s^t = \begin{cases} 1, & \text{if } s = j \\ 0, & \text{if } s \neq j \end{cases}.$$

The model (1.12) implies that the vector of production required to satisfy the demand $\bar{\mathbf{f}}_t$, i.e., $\bar{\mathbf{x}}_t = \mathbf{L}_t \bar{\mathbf{f}}_t = [\bar{x}_s^t, s = 1, \ldots, n]$, is equal to the j-th column in matrix $\mathbf{L}_t$. In other words, l_{ij}^t represents the production of good i, i.e., $\bar{x}_i^t$, that is directly and indirectly needed for each unit of final demand of good j.

1.4.2. Multi-country framework

The principles of constructing single-economy IO models outlined in Section 1.4.1 may be naturally adopted in a multi-national framework; e.g., in the framework of the WIOTs published by WIOD (comp. Figure 1.3). Since I mainly focus on global IO models in further parts of this book, let me assume that there are C groups of countries and S groups of sectors (industries)[25]. Henceforth, I will use the term 'country-sector group' to describe one specific group of sectors operating in one particular group of economies. I assume that each of these country-sector groups produces only one type of goods; therefore, I distinguish $S \cdot C$ types of the final products.

To simplify the notation, I will follow Timmer et al. (2013a) and Gurgul and Lach (2016b, 2019c) in this section and denote i as the origin (source) group of countries, j as the group of destination countries, s as the group of source sectors, and r as the group of destination sectors[26]. The market-clearing conditions imply that the quantity of a set of goods produced in a particular country-sector group equals the domestic and foreign input. Thus, the following identity holds true for each year t:

$$x_i^{s,t} = \sum_{j=1,\ldots,C} f_{ij}^{s,t} + \sum_{j=1,\ldots,C} \left(\sum_{r=1,\ldots,S} x_{ij}^{sr,t} \right), \tag{1.13}$$

where $x_i^{s,t}$ denotes the output in group of sectors s operating in group of countries i, $f_{ij}^{s,t}$ stands for the final demand on these goods in group of countries j in year t, and $x_{ij}^{sr,t}$ stands for the intermediate demand on these goods in group of sectors r operating in the countries listed in group j[27].

One can easily combine the market-clearing conditions (1.13) for each of the $S \cdot C$ goods into a compact aggregated global input-output system. For this purpose, one shall start by denoting $\mathbf{x}_t$ as the $(S \cdot C) \times 1$ vector of output in year t. The latter is obtained by a row-wise concatenation of output levels (each in the form of an $S \times 1$ vector) in each group of countries and takes the following form:

$$\mathbf{x}_t = \begin{bmatrix} \mathbf{x}_1^t \\ \mathbf{x}_2^t \\ \vdots \\ \mathbf{x}_C^t \end{bmatrix}, \quad \mathbf{x}_i^t = [x_i^{s,t}]_{s=1,\ldots,S}, \quad i = 1, \ldots, C. \tag{1.14}$$

Analogously, one may obtain the $(S \cdot C) \times 1$ vector of global final demand (denoted as $\mathbf{f}_t$) by simply stacking world final demand for the output from each

[25] It is clear that all of the methodological remarks in this section hold true for any type of country-sector aggregation, including the case of the lowest possible aggregation (i.e., the case when all countries and all sectors are analyzed separately).

[26] Unlike Section 1.4.1, in this section and in Section 3.4, the symbols i and j will be used to denote countries, not sectors (as is usually done in IO literature). The motivation behind this particular style of notation is to make the content of the two abovementioned sections fully consistent with fundamental papers on the theory and applications of the WIOD database, including the highly-cited groundbreaking work of Timmer et al. (2013a) and a more recent series of papers by Gurgul and Lach (2016b, 2019c, among others).

[27] It is clear that the use of goods can be domestic (in the case of $i = j$) or foreign (when $i \neq j$).

country-sector group. For the sake of clarity, I shall note that final demand $f_i^{s,t}$ will henceforth stand for the sum of the demand for the products of group of sectors s from all groups of countries; i.e.:

$$f_i^{s,t} = \sum_{j=1,\ldots,C} f_{ij}^{s,t}, \quad i = 1,\ldots,C, \quad s = 1,\ldots,S. \tag{1.15}$$

Using this notation, the vector of global final demand takes the following form:

$$\mathbf{f}_t = \begin{bmatrix} \mathbf{f}_1^t \\ \mathbf{f}_2^t \\ \vdots \\ \mathbf{f}_C^t \end{bmatrix}, \quad \mathbf{f}_i^t = [f_i^{s,t}]_{s=1,\ldots,S}, \quad i = 1,\ldots,C. \tag{1.16}$$

Next, one may define an $(S \cdot C) \times (S \cdot C)$ global aggregated intermediate input coefficient matrix $\mathbf{A}_t = [a_{ij}^{sr,t}]_{\substack{i,j=1,\ldots,C \\ s,r=1,\ldots,S}}$ for each period t by the following formula:

$$a_{ij}^{sr,t} = \frac{x_{ij}^{sr,t}}{x_j^{r,t}}, \tag{1.17}$$

where $i, j = 1,\ldots,C$ and $s, r = 1,\ldots,S$. Input coefficient $a_{ij}^{sr,t}$ in (1.17) represents the output from group of sectors s obtained in group of countries i and used as the intermediate input by group of sectors r in the countries listed in group j expressed as a share of output in the latter group of sectors at period t (Gurgul and Lach, 2016b; Timmer et al., 2013a). The elements of matrix $\mathbf{A}_t$ are helpful in answering the question regarding which combinations of various intermediate products (both domestic and foreign) are required to produce one unit of product in each country-sector group. Using the aggregated intermediate input coefficient matrix, one may rewrite the global market clearing conditions (1.13) in the following compact aggregated IO-based form:

$$\mathbf{x}_t = \mathbf{A}_t \mathbf{x}_t + \mathbf{f}_t, \tag{1.18}$$

or equivalently in the form of a demand-driven Leontief model:

$$\mathbf{x}_t = (\mathbf{I} - \mathbf{A}_t)^{-1} \mathbf{f}_t, \tag{1.19}$$

where $\mathbf{I}$ is an $(S \cdot C) \times (S \cdot C)$ identity matrix.

1.5. Closing input-output model

It is important to underline that specification (1.12) defines an 'open' input-output model since output $\mathbf{x}_t$ depends on the existence of an exogenous vector $\mathbf{f}_t$ that, in turn, is disconnected from the technically interrelated productive sectors and whose activity is constituted solely by consumption purchases by households, sales to the government, gross private domestic investment, and shipments in foreign trade. Alternatively, one could move one or more exogenous sectors from the final-demand column and labor input row and place them inside the technically

interrelated table, making them one of the endogenous sectors. In the IO literature, this transformation is known as closing the model with respect to the chosen exogenous sectors.

One can close input-output models with respect to any type of exogenous sectors. Closure with respect to households is the most common case. To construct this type of 'extended' input-output model, a row and column of transactions for the new households sector – the former showing the distribution of its output (labor services) among the various sectors, and the latter showing the structure of its purchases (consumption) distributed among the sectors – are required (Miller and Blair, 2009).

In practical applications, detaching the households-related components of value added and final demand is a challenging task. Besides the issue of the complexity of responses of consumers to changes in income, one should also underline that official labor statistics often do not include data on wages in micro-enterprises that are classified as mixed income (and thus are officially reported as components of the operating surplus). Moreover, a non-zero share of household expenses (including micro-enterprises) is included in the official statistics on gross fixed capital formation (Przybyliński, 2012). On the other hand, the net operating surplus is also used to finance consumer purchases (e.g., through dividend payouts). Another significant practical problem is related to statistical data on imports of households. However, one cannot forget that despite the abovementioned problems with data availability, the closed IO model is the only tool that allows measuring the induced effects in IO multiplier analysis (Chen et al., 2015). To summarize, the motivation to use the closed variant of global IO model in the empirical part of the book follows mainly from the general illustrative purposes, i.e., the main goal is to present the range of possible applications of the new methods of tracing ICs in IO models (including the closed IO models), rather than to formulate actual policy recommendations.

In order to shed some light on the idea of closing the demand-driven model in (1.12) for households, assume once again that the economy under study consists of n productive sectors and the exogenous final demand in each sector is a sum of five components: final consumption expenditure by households, final consumption expenditure by NPISH, final consumption expenditure by government, gross fixed capital formation, and changes in inventories and valuables. In basic static input-output model (1.12), the level of labor income is endogenous in the sense that it reacts to changes in the level and composition of consumption. However, the level of consumption is exogenous in this model (as it is one of the five components of exogenous final demand). Thus, consumption spending does not adjust to reflect the changes in purchasing power when labor requirements change as a consequence of changes in the technical coefficients or in levels of final demand. Due to its exogeneity, the level of consumption changes only if it is adjusted exogenously as part of a particular scenario. Figure 1.6 presents the general structure of an input-output table with an endogenized sector of households.

As shown in Figure 1.6, one could move the households sector from the final-demand column and labor input row and place it inside the technically interrelated table, making it one of the endogenous sectors. In other words, the extended

	Buying Sector					
Selling Sector	**1**	**...**	n	$n+1$ = **Households (consumption)**	**Final demand (household consumption excluded)**	**Output**
1	a^t_{11}	...	a^t_{1n}	$a^t_{1,n+1}$	$\tilde{f}^t_1$	x^t_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	a^t_{n1}	...	a^t_{nn}	$a^t_{n,n+1}$	$\tilde{f}^t_n$	x^t_n
$n+1$ = **Households (labor)**	$a^t_{n+1,1}$	...	$a^t_{n+1,n}$	$a^t_{n+1,n+1}$	$\tilde{f}^t_{n+1}$	x^t_{n+1}
Value added (labor compensation excluded)	$\tilde{v}^t_1$	...	$\tilde{v}^t_n$	$\tilde{v}^t_{n+1}$		
Output	x^t_1	...	x^t_n	x^t_{n+1}		

Figure 1.6. Input-output table with households endogenized

Note:

extended matrix of technical coefficients

elements of non-extended IO table

additional elements in IO table

Source: Own elaboration based on Miller and Blair (2009).

input-output table with households endogenized contains an additional row with data on the labor compensation of households (wages and salaries received by households as payment for their labor services) at period t ($a^t_{n+1,i}$ for $i = 1, \ldots, n+1$ in Figure 1.6) and an additional column with household consumption expenditures at period t ($a^t_{i,n+1}$ for $i = 1, \ldots, n+1$ in Figure 1.6)[28]. The element at the intersection of the additional row and the additional column ($a^t_{n+1,n+1}$) represents household purchases of labor services (e.g., domestic help). In Figure 1.6, $\tilde{f}^t_i$ represents the remaining final demand for sector i's output at period t with household consumption excluded, which is now captured in $a^t_{i,n+1}$[29]. On the other hand, $\tilde{v}^t_i$ includes, for example, other domestic payments and imports in sector i's output at period t with labor compensation excluded. However, due to limited data availability in practical applications, meeting the balance condition (1.18) for the households sector requires imposing some restrictions on the $(n+1)$-th row and the $(n+1)$-th column in the extended IO table depicted in Figure 1.6. Thus, in the empirical part of the book (see the Section 5.3) I will follow the recommendations of Miyazawa (1976), Chen et al. (2015) and Gurgul and Lach (2019c) and nullify the household purchases of labor services (i.e., assume that $a^t_{n+1,n+1} = 0$) and assume that labor is only employed by industries and not in final use (i.e., set $\tilde{f}^t_{n+1} = 0$). Under these

[28] In open input-output model (comp. Figure 1.2 and set of equations (1.12)), the row data on the labor compensation of households is one of the components of the value added row.

[29] Miller and Blair (2009) suggest that $\tilde{f}^t_{n+1}$ would include payments to government employees, for example.

two assumptions the balance condition (1.18) for the households sector, i.e.:

$$\sum_{i=1}^{n+1} a^t_{i,n+1} + \tilde{v}^t_{n+1} = \sum_{i=1}^{n+1} a^t_{n+1,i} + \tilde{f}^t_{n+1},$$

implies that:

$$\tilde{v}^t_{n+1} = \sum_{i=1}^{n} a^t_{n+1,i} - \sum_{i=1}^{n} a^t_{i,n+1},$$

i.e., $\tilde{v}^t_{n+1}$ captures household savings at period t.

After endogenizing the households sector, there would be one new equation for the output of the households sector that is defined to be the total value of its sale of labor services to the various sectors – total earnings. In other words, the extended form of model (1.12) with households endogenized may be formulated in the following way:

$$\mathbf{x}_t = \left(\mathbf{I} - \mathbf{A}^{\text{Extended}}_t\right)^{-1} \tilde{\mathbf{f}}_t, \tag{1.20}$$

where:

$$\tilde{\mathbf{f}}_t = \begin{bmatrix} \tilde{f}^t_1 \\ \vdots \\ \tilde{f}^t_{n+1} \end{bmatrix}, \quad \mathbf{A}^{\text{Extended}}_t = \begin{bmatrix} a^t_{11} & \cdots & a^t_{1,n+1} \\ \vdots & \ddots & \vdots \\ a^t_{n+1,1} & \cdots & a^t_{n+1,n+1} \end{bmatrix}. \tag{1.21}$$

The input coefficients in extended matrix $\mathbf{A}^{\text{Extended}}_t$ are found in the same manner as in the case of the open model, including the elements placed in the additional row and column. In other words, one may use a formula analogous to (1.4). Thus, $a^t_{n+1,j}$ is obtained by dividing the value of sector j purchases of labor for a given period t by the value of the output of sector j for the same period. For the elements of the household purchases column, the $a^t_{i,n+1}$ is calculated as the ratio of value of sector i sales to households for a given period t to the output of the households sector, x^t_{n+1} (Miller and Blair, 2009; Gurgul and Lach, 2019c).

To summarize – the labor row and consumption column are made interdependent in the case of the typical closure for households so that the amount of income governs the consumption outlays and the amount of consumption is a major determinant of the demand for labor and the associated income. This closure assures consistency among the labor requirements, labor income, output, and the level and composition of the consumption – not only for the economy as a whole, but also on a sectoral basis (Duchin and Steenge, 2007).

1.6. Supply-driven input-output model

If only the production technology in an economy under study is known, the demand-driven version of the open input-output model discussed in Section 1.4 allows one to express production as a function of final demand. Ghosh (1958) presented an alternative approach by focusing on the supply-sided version of input-output linkages. In such a context, primary inputs determine output, and producers